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MATRIX REPRESENTATION OF THE GRAPH ADDITION OPERATION

Yakimova Nataliya Anatoliivna
c. t. s., as. prof.
Samkova Galina Evgenivna
c. ph.-math. s., as. prof.
Odessa I.I. Mechnikov National University, str. Dvoryanska, 2, Odessa, Ukraine, 65029

Abstract. *Graph theory is widely used from a practical point of view. Graphs surround us in everyday life (for example, maps of roads and paths), and also play an important role in scientific research (for example, electrical circuits). For everyday use, the geometric way of presenting graphs is certainly the most convenient. But for computer processing of information, this is not rational. In these cases, the matrix representation of graphs is used. Therefore, studies devoted to this topic are gaining more and more importance. This article considers the possibility and methods of performing operation of addition of both directed and undirected graphs.*

Keywords: *directed and undirected graph, adjacency matrix, operation of addition of the graphs, elementary logical operations, Boolean matrix, multivalued logic.*

Introduction. If the matrices are Boolean, then with them it is possible to perform both ordinary algebraic operations on matrices and two-valued logic operations described in [1]. If the matrices are not Boolean, then in order to perform the logical operations of disjunction and conjunction with them, it is necessary to use the apparatus of multi-valued logic. In this case, the operations of disjunction and conjunction of matrices are performed according to the following rules [2]:

$$x \vee y = \max\{x, y\}, \tag{1}$$

$$x \wedge y = \min\{x, y\}, \tag{2}$$

Main text.

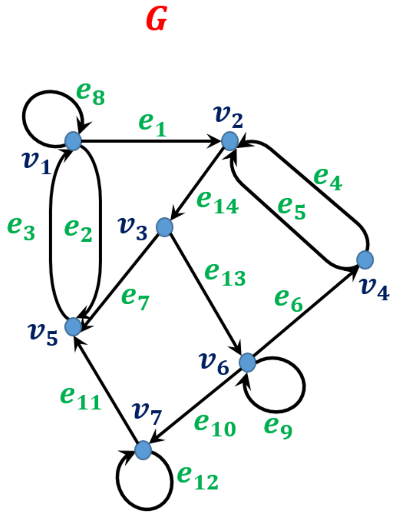

Figure 1 – Directed graph

Author's development

The possibility of matrix execution of basic operations on graphs can be shown on examples. Consider the directed graph shown in figure 1.

Adjacency matrices can be constructed for this graph:

$A(G) =$

		Final						
		v_1	v_2	v_3	v_4	v_5	v_6	v_7
Initial	v_1	1	1	0	0	1	0	0
	v_2	0	0	1	0	0	0	0
	v_3	0	0	0	0	1	1	0
	v_4	0	2	0	0	0	0	0
	v_5	1	0	0	0	0	0	0
	v_6	0	0	0	1	0	1	1
	v_7	0	0	0	0	1	0	1

or

$$A(G) = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

This matrix are constructed assuming that the row number corresponds to the starting vertex and the column number corresponds to the final vertex of each edge. But this graph can be defined by this matrix from the very beginning. Performing operations on such graph does not require reproduction of its geometric implementation.

Graph G contains strictly parallel edges $e_4(v_4, v_2)$ and $e_5(v_4, v_2)$. Therefore, its adjacency matrix contains the element a_{42}=2, i.e. it is not Boolean.

The *addition operation* is defined for undirected graphs. Therefore, if the graph is directed or mixed, first it is necessary to build an associated (or correlated) graph for it [3], and then perform the addition operation for this new graph. From the adjacency matrix $A(G)$ of the initial graph, we calculate the new matrix $A^s(G)$ according to the following rule:

$$a_{ij}^s = a_{ji}^s = \min\{1, \max\{a_{ij}, a_{ji}\}\}. \tag{3}$$

The resulting matrix will be Boolean. But it can contain units on the main diagonal, that is, display a graph with loops. A correlated graph, which cannot contain loops, is used in these calculations only as an intermediate result. Therefore, you can get rid of these loops once already at the last step, and not every time for all intermediate matrices. The next step to obtain the adjacency matrix of the addition of the graph G is to perform the logical operation of the inversion [1] of the matrix $A^s(G)$, that is, to calculate the matrix $\overline{A^s(G)}$.

For example, let's find the adjacency matrix of the addition of the directed graph G. Let's construct the matrix $A^s(G)$ for this graph:

$$A^s(G) = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

Now let's calculate the inversion of this matrix, that is, the matrix $\overline{A^s(G)}$:

$$\overline{A^s(G)} = \begin{pmatrix} \overline{1} & \overline{1} & \overline{0} & \overline{0} & \overline{1} & \overline{0} & \overline{0} \\ \overline{1} & \overline{0} & \overline{1} & \overline{1} & \overline{0} & \overline{0} & \overline{0} \\ \overline{0} & \overline{1} & \overline{0} & \overline{0} & \overline{1} & \overline{1} & \overline{0} \\ \overline{0} & \overline{1} & \overline{0} & \overline{0} & \overline{0} & \overline{1} & \overline{0} \\ \overline{1} & \overline{0} & \overline{1} & \overline{0} & \overline{0} & \overline{0} & \overline{1} \\ \overline{0} & \overline{0} & \overline{1} & \overline{1} & \overline{0} & \overline{1} & \overline{1} \\ \overline{0} & \overline{0} & \overline{0} & \overline{0} & \overline{1} & \overline{1} & \overline{1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Now it is necessary to get rid of loops in the obtained graph, that is, units on the main diagonal of the obtained matrix. For this, it is necessary to perform its conjunction with the inversion of the unit matrix. As a result, the addition graph adjacency matrix will be obtained:

$$\overline{A^s(G)}\wedge\overline{E}=\begin{pmatrix}0&0&1&1&0&1&1\\0&1&0&0&1&1&1\\1&0&1&1&0&0&1\\1&0&1&1&1&0&1\\0&1&0&1&1&1&0\\1&1&0&0&1&0&0\\1&1&1&1&0&0&0\end{pmatrix}\wedge\begin{pmatrix}0&1&1&1&1&1&1\\1&0&1&1&1&1&1\\1&1&0&1&1&1&1\\1&1&1&0&1&1&1\\1&1&1&1&0&1&1\\1&1&1&1&1&0&1\\1&1&1&1&1&1&0\end{pmatrix}=$$

$$=\begin{pmatrix}0&0&1&1&0&1&1\\0&0&0&0&1&1&1\\1&0&0&1&0&0&1\\1&0&1&0&1&0&1\\0&1&0&1&0&1&0\\1&1&0&0&1&0&0\\1&1&1&1&0&0&0\end{pmatrix}=A(\overline{G}).$$

For undirected graphs, formula (3) takes the form

$$a_{ij}^s=a_{ji}^s=\min\{1,a_{ij}\}\,. \qquad (4)$$

For undirected graphs without multiple edges, the adjacency matrix of the addition graph can be obtained immediately by the conjunction of the inversion of the adjacency matrix of the original graph with the matrix $\overline{E}$.

In Boolean algebra, the conjunction with one does not affect the result. Therefore, in the program implementation, at the last step of constructing the matrix $A(\overline{G})$, you can simply assign a zero value to all elements of the main diagonal, that is, to all elements for which the column number and the row number (*i*=*j*) coincide.

Summary and conclusions.

The same algorithms have differences depending on whether directed or free graphs are involved in the considered operations. Depending on the types of graphs, there are also restrictions on the display of meaningful information by the matrices of these graphs. But in practical applications, these restrictions are usually insignificant. Therefore, for each operation on graphs and each type of graph, it is possible to propose a combination of algebraic operations (arithmetic and logical) that allow obtaining the matrix of a new graph, or a clear, easily programmable algorithm for transforming the matrices of the initial graphs.

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