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TRANSIENT DYNAMICS OF WAVE PROPAGATION IN HETEROGENEOUS COMPOSITES

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Abstract. *This paper is devoted to the development of a local homogenization model for multiscale heterogeneous composite materials. The key approach of this model is a continuous wavelet representation of the composite material properties and the corresponding multiscale reduction of parameters that describe the volume averaging of the reduction methodology. Numerical experiments performed based on the assumptions of this model indicate that the classical homogenization method previously used for two-scale composites, as well as for composites with a scale variable in a wide range, is a special case of the general multiscale strategy, where one scale parameter tends to zero. Additional analysis was performed for unidirectional wavelet homogenization of a linear elastic heterogeneous problem and for the dynamics of wave propagation. It is shown that wavelet homogenization can be used in combination with various discrete numerical methods for efficient modeling of heterogeneous solids, liquids and multiphase media.*

Key words: *local homogenization, composite materials, wavelet representation, multiscale reduction, wave propagation.*

Introduction.

One of the most frequently used approaches to computational modeling of both mechanical and thermodynamic properties of the volume of composite systems is the homogenization method [1]. It should be noted that the basic assumption in such methodologies is the postulate of the existence of a fixed and analytically specified scale relationship between the components of the composite and the entire system. Most often, numerical analysis is based on two-scale models that are related by a scale parameter that is a small real value (usually a negligible value) [2].

However, the use of such a methodology is associated with a significant drawback, namely, the impossibility of considering more than two different scales in the volume of the composite structure. In addition, the sensitivity of the homogenized characteristics of the composite to the interrelations of geometric scales cannot be specified with the required accuracy [3]. To summarize, it can be stated that the multiscale methodology based on wavelet analysis is a fairly popular technique in signal theory. Multiscale homogenization allows analyzing composite systems with several geometric scales. The results of such analysis, performed for most laminated composites, allow a detailed description of the scale of microdefects and their further development.

Wavelet analysis is often used as a subsequent step in the development of the homogenization model, which is a particularly useful tool in the field of composite materials. Wavelet analysis allows direct analysis of multiscale heterogeneous structures using fixed-type wavelets [4]. In addition, multidimensional decomposition of the spatial distribution of composite materials and their physical properties is only possible using different types of wavelets that are adapted to specify different scales. The use of wavelet transforms in homogenization procedures allows the analysis of both experimental results (analysis of composite morphology images) and the results of numerical approximations.

Multiresolutional homogenization.

The procedure of multiscale homogenization of the volume of composite structures in this model is applied to all ordinary differential equations, which can be written as

$$Bx + q + \lambda = K(Ax + p), \tag{1}$$

where A, B are the wavelet transforms for the parent functions W-W and V-W, respectively;

x, p, q, λ are the fixed coordinate, terms of force, shear and mechanical stress, respectively;

K is the Haar basis element.

The following recursive relations can be written to specify the scales of local volumes in the j-th layer of the composite

$$\left(A_{k+1}^{(j)}\right) = (S_A)_i - (D_A)_i F^{-1}\left[(D_B)_i + \frac{\delta_k}{2}(S_A)_i\right], \tag{2}$$

$$\left(B_{k+1}^{(j)}\right)_i = (S_B)_i - \frac{\delta_k}{2}(D_A)_i - \left[(D_B)_i - \frac{\delta_k}{2}(S_A)_i\right]F^{-1}\left[(D_B)_i + \frac{\delta_k}{2}(S_A)_i\right], \tag{3}$$

where $\delta_k = 2^{-k}$,

Ψ_h, $\Psi \in (S, D, F)$, $h \in (A, B)$ are the Haar transforms of the first, second and third order, respectively.

The recurrence relations written earlier are local and can be satisfied on as many scales as necessary. It is assumed that the functional F^{-1} exists on each scale. In this case, the following relation can be written

$$B_0^{(j)} x_0^{(j)} + q_0^{(j)} + \lambda = K_0 \left(A_0^{(j)} x_0^{(j)} + p_0^{(j)} \right), \tag{4}$$

where the recurrence relations is applied j times to compute $B_0^{(j)}$, $A_0^{(j)}$, $p_0^{(j)}$,$q_0^{(j)}$.

It should be noted that this homogenization procedure allows the coefficients to vary over an arbitrary number of intermediate scales. This difference is an advantage over classical homogenization examples that did not allow any intermediate scales.

The implications for the homogenization model can be divided into two sets. The first set of results concerns the possibility of establishing a general structure for multiscale reduction and homogenization. The second set of results indicates that the use of a Haar basis (or multiwavelet basis) for systems of linear ordinary differential equations provides significant computational convenience. Since the Haar functions at a fixed scale do not have overlapping supports, the recurrence relations for the operators and forcing terms in the equation can be written as local relations and solved explicitly.

The multiscale analysis model included a system of ordinary differential equations that corresponded to the propagation of unidirectional acoustic waves in a multiscale medium with a uniaxial strain field. The multiscale distribution of inhomogeneities is specified using the relation

$$\frac{du(x)}{dx} = i\omega M(x) u(x); \qquad x \in [0, 1], \tag{5}$$

where the coefficients M (x), which are defined in the Haar wavelet approximation for the entire set of composite layers used, satisfy the following equation

$$M(x) = \begin{cases} M_0, \ 0 \le x < 1/2 \\ M_1, \ 1/2 \le x \le 1 \end{cases}. \tag{6}$$

Summary and conclusions.

The multiscale homogenization method allows an alternative approach whereby the effective parameters of the composite material are first determined and then the entire composite is analyzed using traditional computational methods. The Haar wavelet transforms applied in the multiscale homogenization model allow the characteristics of the components at many scales to be incorporated into the final effective structural parameters. The homogenized properties in the multiscale analysis and the classical macro-micro pass differ significantly, even in the deterministic formulation, as was previously observed in three-scale homogenization studies based on Monte Carlo simulations for fiber-reinforced composites.

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